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International Journal of Solids and Structures 41 (2004) 3643–3652

INTERNATIONAL JOURNAL OF SOLIDS and STRUCTURES

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Identification of an impulse load acting on an axisymmetrical hemispherical shell

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Received 14 January 2003; received in revised form 30 September 2003
Available online 11 March 2004

Abstract

The paper presents the results of solving the problem of identifying a non-stationary surface load acting on an axisymmetrical hemispherical shell with a rigidly fastened edge. The inverse problem is solved by using the non-classical theory of shells and Tikhonov's regularization method.

Keywords: Inverse problem; Impulse loading; Hemispherical shell

1. Introduction

The results of solving the inverse problems in the mechanics of a deformed solid find wide application, e.g. in exploratory design, when refined data on external non-stationary loads acting on structural components during their service are required. Availability of such information allows to effectively solve design problems due to the possibility of in-depth investigation of the deflected mode of a mechanical system, thereby ensuring the required reliability and durability of the structure. This problem is especially crucial for components of structures operating under conditions of non-stationary impulse loads of the shock and explosive action type.

At present, effective method of solving inverse problems in identifying external force actions are being intensively developed. At this, the source data are values, which are indirect manifestations of the sought for functions, and which can be obtained experimentally. The solutions of non-stationary problems of such kind for rods are known in the literature (Krasnobaev and Potyetiunko, 1989; Lukianova, 1985; Romanenko et al., 1989; Gladwell, 1984). Problems of such type for shells have been investigated far less, and in the first place these are static problems. Let us mention first the works of Kylatchanov (1988) and Tarasiuk (1993), which present the results of identifying an external action based on non-linear defining

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0020-7683/$ - see front matter
doi:10.1016/j.ijsolstr.2003.10.009

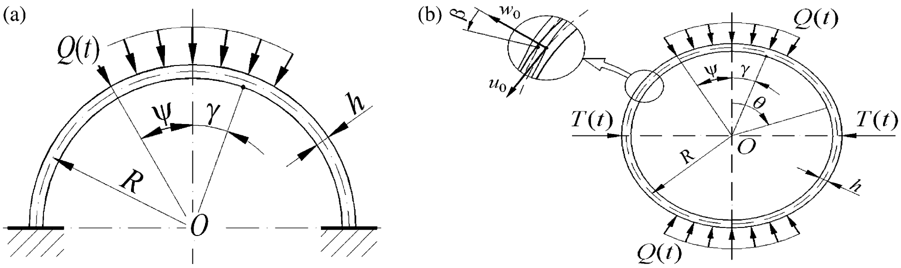

Fig. 1. The investigated mechanical system (a) and the equivalent one (b).

equations for anisotropic media. The identification of a dynamic distributed load acting on a cylindrical shell at non-axisymmetrical loading has been discussed in the paper (Yanyutin and Yanchevsky, 2000).

This paper presents the solution of the problem of identifying a non-stationary load acting on a hemispherical shell with a rigidly fixed edge, the shell loading area being defined by the apex angle ψ in the spherical coordinate system (Fig. 1a). Since this problem relates to the class of ill-posed problems in mathematical physics, Tikhonov's regularization method was used to find a stable solution. The paper also presents the results of comparing the solution of the identification problem with experimental data in the literature.

2. Direct problem

At the first stage of solving the inverse problem being investigated, we build the solution for a respective straight line, whose solution results in a relationship between the experimental function $\varepsilon_\varphi(\gamma, t)$ and the load $Q(t)$ being identified. The solution is based on the method of Yanyutin (1993), which consists in considering a closed spherical shell, assuming the symmetry of its strain with respect to the equatorial plane ($\theta = \pi/2$). In so doing, the equality to zero of the tangential displacement of the median surface u_0 and of the angle of rotation of the normal in the meridian plane β in point $\theta = \pi/2$ holds. Meeting the missing boundary condition ($w_0(\pi/2, t) = 0$, where w_0 is the radial displacement) is ensured by applying an additional lumped normal load $T(t)$ (Fig. 1b) to the shell in this point, the load being found when constructing the solution of the direct problem.

By virtue of the accepted assumptions, the closed spherical shell is effected by the total normal load (Fig. 1b)

$$N(\theta, t) = Q(t) \cdot H(|\cos\theta| - |\cos\psi|) + T(t) \cdot \delta(\theta - \pi/2) \quad (\theta \in [0, \pi]),$$

where $H(\theta)$ is Heaviside's unit function; and $\delta(\theta)$ is the delta-function.

In the theory of the Timoshenko type, the equations of motion for a spherical shell at its axisymmetrical deformation have the form (Yanyutin, 1993)

$$\left.\begin{aligned}
&\alpha_1\left[(1 - z^2)\frac{\partial^2 u_0}{\partial z^2} - 2z\frac{\partial u_0}{\partial z} - \frac{\nu + (1-\nu)z^2}{1 - z^2}u_0\right] - \alpha_2\sqrt{1 - z^2}\frac{\partial w_0}{\partial z} + \alpha_3\beta = \frac{\partial^2 u_0}{\partial \tau^2};\\
&\alpha_4\left[(1 - z^2)\frac{\partial^2 w_0}{\partial z^2} - 2z\frac{\partial w_0}{\partial z}\right] - \alpha_5 w_0 - \alpha_6\left[-\sqrt{1 - z^2}\frac{\partial u_0}{\partial z} + \frac{z u_0}{\sqrt{1 - z^2}}\right]\\
&+\alpha_3\left[-\sqrt{1 - z^2}\frac{\partial \beta}{\partial z} + \frac{z\beta}{\sqrt{1 - z^2}}\right] = \frac{\partial^2 w_0}{\partial \tau^2} - q_0 N(z, \tau);\\
&\alpha_7\left[(1 - z^2)\frac{\partial^2 \beta}{\partial z^2} - 2z\frac{\partial \beta}{\partial z} - \frac{\nu + (1-\nu)z^2}{1 - z^2}\beta\right] - \alpha_8\beta + \alpha_9\sqrt{1 - z^2}\frac{\partial w_0}{\partial z} = \frac{\partial^2 \beta}{\partial \tau^2},
\end{aligned}\right\} \tag{1}$$

where $\tau = t\sqrt{\frac{E}{\rho R^2(1-\nu^2)}}$ is dimensionless time; $z = \cos\theta$;

$$\alpha_1 = \frac{1}{k_1}; \quad \alpha_2 = \frac{1}{k_1}\left(1 + v + \frac{1-v}{2k_s}\right); \quad \alpha_3 = \frac{R(1-v)}{2k_s k_1}; \quad \alpha_4 = \frac{1-v}{2k_s k_1}; \quad \alpha_5 = \frac{2(1+v)}{k_1};$$
$$\alpha_6 = \frac{1+v}{k_1}; \quad \alpha_7 = \frac{1}{k_r}; \quad \alpha_8 = \frac{6R^2(1-v)}{k_s k_r h^2}; \quad \alpha_9 = \frac{6R(1-v)}{k_s k_r h^2}; \quad q_0 = \frac{R^2(1-v^2)}{Ek_1 h};$$

E, v, ρ are elastic constants and the shell material density; R, h are the median surface radius and the shell thickness; k_s^2 is the shear coefficient; $k_1 = 1 + h^2/(12R^2)$; $k_r = 1 + 3h^2/(20R^2)$.

With account of zero initial conditions, the solution of the system of equations (1) takes the form (Yanyutin, 1993)

$$\left.\begin{aligned} & w_0(z,\tau) = \textstyle\sum_{n=0}^{\infty} a_{2n}(\tau)P_{2n}(z); \\ & \begin{Bmatrix} u_0(z,\tau) \\ \beta(z,\tau) \end{Bmatrix} = \textstyle\sum_{n=1}^{\infty} \begin{Bmatrix} b_{2n}(\tau) \\ c_{2n}(\tau) \end{Bmatrix} P^1_{2n}(z), \end{aligned}\right\} \tag{2}$$

where $a_0(\tau) = \frac{q_0}{\sqrt{\alpha_5}} \int_0^\tau V_0(\chi) \cdot \sin(\sqrt{\alpha_5}(\tau - \chi))\,\mathrm{d}\chi$;

$$(\zeta)_{2n}(\tau) = q_0(4n+1)\sum_{r=1}^{3} \frac{\Lambda^{\zeta}_{2n,r}}{\alpha_{2n,r}\prod_{j=1 j\neq r}^{3}(\alpha^2_{2n,j} - \alpha^2_{2n,r})} \int_0^\tau V_{2n}(\chi) \cdot \sin(\alpha_{2n,r}(\tau - \chi))\,\mathrm{d}\chi \quad (\zeta = a, b, c);$$

$$V_{2n}(\chi) = Q(\chi)\int_{\cos\psi}^{1} P_{2n}(z)\,\mathrm{d}z + \frac{1}{2R}T(\chi)P_{2n}(0) \quad (n = 0, 1, 2, \ldots);$$

$$\Lambda^a_{2n,r} = (A_{2n} - \alpha^2_{2n,r})(C_{2n} - \alpha^2_{2n,r}); \quad \Lambda^b_{2n,r} = \alpha_2(C_{2n} - \alpha^2_{2n,r}) - \alpha_3\alpha_9; \quad \Lambda^c_{2n,r} = -\alpha_9(A_{2n} - \alpha^2_{2n,r}),$$

at this

$$A_{2n} = \alpha_1[2n(2n+1) - (1-v)]; \quad B_{2n} = 2n\alpha_4(2n+1) + \alpha_5; \quad C_{2n} = \alpha_7[2n(2n+1) - (1-v)] + \alpha_8.$$

Let us note that in relationships (2) $P_n(z)$, $P_n^1(z)$ are Legendre's associated functions, and values $\alpha_{2n,r}(r = 1, 2, 3)$ are the modules of the roots of a binary cubic equation

$$s^6 + e_{2n}s^4 + d_{2n}s^2 + m_{2n} = 0,$$

where

$$e_{2n} = A_{2n} + B_{2n} + C_{2n};$$

$$d_{2n} = A_{2n}B_{2n} + B_{2n}C_{2n} + A_{2n}C_{2n} - 2n(2n+1)(\alpha_2\alpha_6 + \alpha_3\alpha_9);$$

$$m_{2n} = A_{2n}B_{2n}C_{2n} - 2n(2n+1)(\alpha_2\alpha_6 C_{2n} + \alpha_3\alpha_9(A_{2n} - \alpha_6)).$$

The unit strain $\varepsilon_\varphi(\gamma, t)$ on the inner surface of the shell is written in the form (Filin, 1987)

$$\varepsilon_\varphi(\gamma, t) = \frac{1}{R - h/2}\left(w_0(\gamma, t) + \mathrm{ctg}(\gamma)\left(u_0(\gamma, t) - \frac{h}{2}\beta(\gamma, t)\right)\right), \tag{3}$$

it being considered as source data when solving the inverse problem.

After substituting expansions (2) into (3), we obtain a relationship, which can be written down in operator form as

$$A_{QE}Q + A_{TE}T = E.$$

The force $T(t)$, involved in this expression, is found from the condition $w_0(\pi/2, t) = 0$, whose operator form of expression can be presented as

$$A_{Q0}Q + A_{T0}T = \mathbf{0}.$$

The operators $A_{QE}Q$, $A_{Q0}Q$, $A_{TE}T$ and $A_{T0}T$ introduced earlier, denote the following, respectively:

$$A_{QE}Q \equiv \int_0^\tau Q(\chi)\left((1-\cos\psi)\frac{\sin\left(\sqrt{\alpha_5}(\tau-\chi)\right)}{\sqrt{\alpha_5}} + \sum_{n=1}^{\infty}\int_{\cos\psi}^{1} P_{2n}(z)\,\mathrm{d}z \sum_{r=1}^{3}\Omega(\gamma)_{2n,r}\sin(\alpha_{2n,r}(\tau-\chi))\right)\mathrm{d}\chi;$$

$$A_{TE}T \equiv \int_0^\tau T(\chi)\frac{1}{2R}\left(\frac{\sin\left(\sqrt{\alpha_5}(\tau-\chi)\right)}{\sqrt{\alpha_5}} + \sum_{n=1}^{\infty}P_{2n}(0)\sum_{r=1}^{3}\Omega(\gamma)_{2n,r}\sin(\alpha_{2n,r}(\tau-\chi))\right)\mathrm{d}\chi;$$

$$A_{Q0}Q \equiv \int_0^\tau Q(\chi)\left((1-\cos\psi)\frac{\sin(\sqrt{\alpha_5}(\tau-\chi))}{\sqrt{\alpha_5}} + \sum_{n=1}^{\infty}\int_{\cos\psi}^{1} P_{2n}(z)\,\mathrm{d}z \sum_{r=1}^{3}\Omega^{*}_{2n,r}\sin(\alpha_{2n,r}(\tau-\chi))\right)\mathrm{d}\chi;$$

$$A_{T0}T \equiv \int_0^\tau T(\chi)\frac{1}{2R}\left(\frac{\sin(\sqrt{\alpha_5}(\tau-\chi))}{\sqrt{\alpha_5}} + \sum_{n=1}^{\infty}P_{2n}(0)\sum_{r=1}^{3}\Omega^{*}_{2n,r}\sin(\alpha_{2n,r}(\tau-\chi))\right)\mathrm{d}\chi,$$

where

$$\Omega(\gamma)_{2n,r} = (4n+1) \times \frac{\left[P_{2n}(\cos\gamma)(A_{2n}-\alpha^2_{2n,r})(C_{2n}-\alpha^2_{2n,r}) + P^1_{2n}(\cos\gamma)\mathrm{ctg}(\gamma)\left[(\alpha_2(C_{2n}-\alpha^2_{2n,r}) - \alpha_3\alpha_9) + \frac{\alpha_9 h}{2}(A_{2n}-\alpha^2_{2n,r})\right]\right]}{\alpha_{2n,r}\prod_{j=1,j\neq r}^{3}(\alpha^2_{2n,j}-\alpha^2_{2n,r})};$$

$$\Omega^{*}_{2n,r} = (4n+1)P_{2n}(0)\frac{(A_{2n}-\alpha^2_{2n,r})(C_{2n}-\alpha^2_{2n,r})}{\alpha_{2n,r}\prod_{j=1,j\neq r}^{3}(\alpha^2_{2n,j}-\alpha^2_{2n,r})},$$

and function $E(\tau) = \varepsilon_\varphi(\gamma,\tau)\cdot(R-h/2)/q_0$.

3. Inverse problem

As pointed out earlier, the solution of the inverse problem can be reduced to finding the approximate solution of a system of integral Volterra equations of the 1st kind with a known right part:

$$\left.\begin{aligned} A_{QE}Q + A_{TE}T &= E; \\ A_{Q0}Q + A_{T0}T &= \mathbf{0}. \end{aligned}\right\} \tag{4}$$

Note that the problem of numerical solution of systems of type (4) with completely continuous operators and perturbed source data is an ill-posed one. To solve such systems, it is necessary to apply special methods, the key one amongst others, due to its several advantages, being Tikhonov's regularization method (Tikhonov et al., 1990). Its implementation in this case consists in preliminary discretization of the initial system (4). For this, in the time interval $[0, \tau_{\mathrm{inv}}]$ being investigated, we select a uniform grid with the step $\Delta\tau$ and introduce the following designations:

$$Q_p = Q(p\cdot\Delta\tau);\quad T_p = T(p\cdot\Delta\tau);\quad E_m = E(m\cdot\Delta\tau);\quad p = 0,1,2,\ldots,m_{\max}-1;$$
$$m = 1,2,\ldots,m_{\max};\quad m_{\max} = \tau_{\mathrm{inv}}/\Delta\tau.$$

Then in system (4), Q, T and E are $m_{\max}$-dimensional vector-columns; A_{QE}, A_{TE}, A_{Q0} and A_{T0} are the finite-difference analogs of the respective operators (the bottom triangular $(m_{\max}\times m_{\max})$-matrices). The values of the components of the latter are defined by the selected quadrature formula used for substituting the integrals involved therein with finite sums (the trapezoids formula was used in this paper). For example, the components of matrix A_{QE} are defined by the formula:

$$A_{QE_{m,p+1}} \equiv \begin{cases} \omega_p\Big((1-\cos\psi)\frac{\sin(\sqrt{\alpha_5}(m-p)\Delta\tau)}{\sqrt{\alpha_5}} + \sum_{n=1}^{\infty}\int_{\cos\psi}^{1} P_{2n}(z)\,\mathrm{d}z \sum_{r=1}^{3}\Omega(\gamma)_{2n,r}\sin(\alpha_{2n,r}(m-p)\Delta\tau)\Big) \\ \quad \text{for } m > p; \\ 0 \quad \text{for } m \leqslant p, \end{cases}$$

where $\omega_0 = 0.5$; $\omega_p = 1$ $(p \neq 0)$ are the weights of the trapezoids quadrature formula.

The components of the remaining matrices are computed similarly.

In view of the fact that only vector Q is sought for, Tikhonov's regularization method (Tikhonov et al., 1990) is applied to the system of linear algebraic equations (SLAE)

$$AQ = E, \tag{5}$$

where $A = A_{QE} - A_{TE}(A_{T0})^{-1}A_{Q0}$ is the matrix $(m_{\max} \times m_{\max})$, to which the initial system (4) is reduced by elementary mathematical manipulations.

As is well known, the solution of the problem of minimizing the smoothing parametric functional in Tikhonov's method is equivalent to solving a SLAE of the form

$$(A^{\mathrm{T}}A + \alpha C)Q = A^{\mathrm{T}}E, \tag{6}$$

where A^{T} is a matrix transposed to A; $\alpha > 0$ is the regularization parameter; C is a symmetrical tridiagonal $(m_{\max} \times m_{\max})$-matrix in the following form:

$$C = \begin{bmatrix} 1+\Delta\tau^{-2} & -\Delta\tau^{-2} & 0 & \dots & 0 & 0 \\ -\Delta\tau^{-2} & 1+2\Delta\tau^{-2} & -\Delta\tau^{-2} & \dots & 0 & 0 \\ 0 & -\Delta\tau^{-2} & 1+2\Delta\tau^{-2} & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 1+2\Delta\tau^{-2} & -\Delta\tau^{-2} \\ 0 & 0 & 0 & \dots & -\Delta\tau^{-2} & 1+\Delta\tau^{-2} \end{bmatrix}.$$

In so doing, it is assumed that the sought for function is continuous in the time interval being investigated, and that almost everywhere it has a square-integrated derivative.

The value of parameter α can be computed using the residual principle (Tikhonov et al., 1990), which consists in matching the residual value for the regularized solution with the error level in the source data. This principle can be written as

$$\|E - AQ\| = \kappa\|E\|, \tag{7}$$

where $\kappa \in (0; 1]$ is the relative residual factor (its value is the greater the greater the error).

Hence, α is defined so that the corresponding thereto vector Q, as the solution of the SLAE (6), would ensure satisfying of Eq. (7). We note that an effective method of solving (6), with account of condition (7), is described in the work of Gordonova (1973).

An alternative value of coefficient κ_{apt}, and the corresponding thereto value of parameter α_{apt}, can be determined by analyzing for "saw-toothness" the temporal behavior of function $Q(t)$, as a solution of the SLAE (6), which satisfies condition (7) with different κ. Having set the initial κ_1 equal, for instance, to 0.01, the subsequent values of κ_n $(n = 2, 3, \dots n_{\max})$ for eliminating the indicated behavior of load $Q(t)$ are chosen by increasing the previous one by the value of the selected step $\Delta\kappa_n$, i.e. $\kappa_{n+1} = \kappa_n + \Delta\kappa_n$. The alternative value of κ_{apt}, and hence, of α_{apt}, is selected so as to reduce the level of "saw-toothness", retaining at the same time the physical content of the load function, which is controlled by proper selection of the value $n_{\max}$ $(\kappa_{\mathrm{apt}} = \kappa_{n_{\max}+1})$.

4. Numerical results

The following shell parameters have been accepted:

$$R = 0.3\ \text{m};\quad h = 0.03\ \text{m};\quad E = 2.1\times 10^{11}\ \text{Pa};\quad \rho = 7800\ \text{kg/m}^3;\quad \nu = 1/3;\quad k_s^2 = 6/5.$$

The "noisy" function $\tilde{\varepsilon}_\varphi(\gamma,\tau) = \bar{\varepsilon}_\varphi(\gamma,\tau)\cdot(1 + \Delta\cdot r(\tau))$ was accepted as the source data, where $\bar{\varepsilon}_\varphi(\gamma,\tau)$ is the result of solving the direct problem at a known load $\overline{Q}(\tau)$; Δ is the relative error, $\Delta = 0.05$; and $r(\tau)$ is the random numbers function with the mathematical expectation in zero, $r(\tau)\in[-1;1]$.

The values of other design parameters are as follows: $\tau_{\text{inv}} = 15.71$; $\Delta\tau = 0.209$; $\psi = \pi/4$; and $\gamma = \pi/6$.

Fig. 2 shows the unit strain $\bar{\varepsilon}_\varphi(\theta,\tau)$ vs. time curve in point $\theta = \gamma$ at $\overline{Q}(\tau) = H_0\cdot(H(\tau) - H(\tau - 2\pi))$ ($H_0 = 10^5$ H/m^2 is the load intensity), which was accepted as the source data for the identification problem, and which was obtained when solving the respective direct problem. In Fig. 3, the dashed line represents the exact values of the sought for function $\overline{Q}(\tau)$, whereas the solid line represents the function $Q(\tau)$ computed from system (6) with account of (7) at $\kappa = \kappa_{\text{apt}}$.

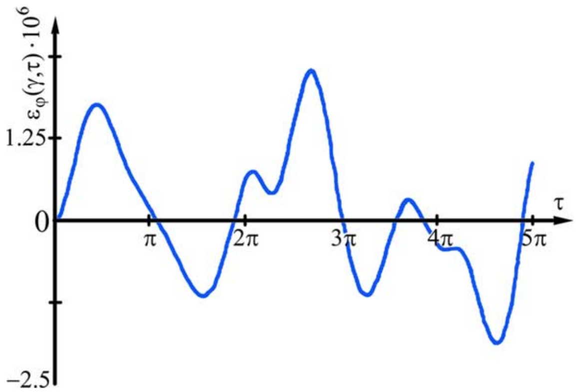

Fig. 2. Source data for solving the identification problem.

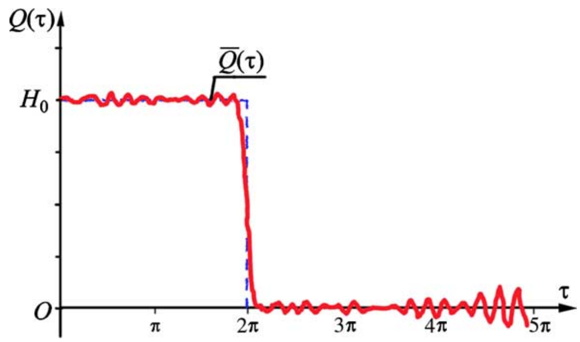

Fig. 3. Result of solving the inverse problem (solid curve).

5. Comparison with experimental data

Of interest is the comparison of results obtained for identification of loading with experimental investigations data. In the monograph (Vorobiev et al., 1989), an experimental investigation into non-stationary strain of a hemispherical shell with a rigidly fixed edge is described. Elastic impact of steel balls dropped from various heights with a shell was considered. To investigate the process of propagation of strain over the shell meridian, several sensors were installed on the shell inner surface (the 1st sensor was installed in the pole, and the remaining ones were installed along the meridian with a 9 mm step between the installation points). The measurement results were presented in the form of oscillograms, allowing to build the stress distribution diagrams for the coordinate angle θ at different points of time.

The procedure of comparing the theoretical design of this work with experimental results shown in Fig. 78 in Vorobiev et al. (1989) comprised two stages. The first stage, based on one of the stress oscillograms, consists in defining (identifying) the contact load in the impact zone. In particular, we chose the oscillogram corresponding to the designation "18 mm". The second stage consists in comparing the experimental results, defined by all the remaining oscillograms, with the theoretical results of solving the direct problem of impulse strain in a shell under the effect of the known load.

The parameter recorded in work (Vorobiev et al., 1989) in point $\theta = \gamma$ on the shell's inner surface defines the unit strain value along the meridian, which is written as (Filin, 1987)

$$\varepsilon_\theta(\tau) = \frac{1}{R - h/2}\left(w_0(\gamma,\tau) + \frac{\partial}{\partial\theta}\left(u_0(\gamma,\tau) - \frac{h}{2}\beta(\gamma,\tau)\right)\right).$$

As applied to the problem considered here, after straightforward manipulations with account of (2), we obtain the following expression for $\varepsilon_\theta(\gamma,\tau)$:

$$\varepsilon_\theta(\gamma,\tau) = \frac{q_0}{R - h/2}\int_0^\tau \sum_{n=1}^{\infty} V_{2n}(\chi)\sum_{r=1}^{3}\Omega^{**}(\gamma)_{2n,r}\sin(\alpha_{2n,r}(\tau-\chi))\,\mathrm{d}\chi, \tag{8}$$

where

$$\Omega^{**}(\gamma)_{2n,r} = (4n+1) \times \frac{\left[P_{2n}(\cos(\gamma))(A_{2n}-\alpha_{2n,r}^2)(C_{2n}-\alpha_{2n,r}^2) + (P_{2n}^2(\cos(\gamma)) + P_{2n}^1(\cos(\gamma))\mathrm{ctg}(\gamma))\left[(\alpha_2(C_{2n}-\alpha_{2n,r}^2) - \alpha_3\alpha_9) + \frac{\alpha_9 h}{2}(A_{2n}-\alpha_{2n,r}^2)\right]\right]}{\alpha_{2n,r}\prod_{j=1,j\neq r}^{3}(\alpha_{2n,j}^2 - \alpha_{2n,r}^2)}.$$

The values of the remaining coefficients included in (8) were given earlier.

Defining functions $Q(\tau)$ and $T(\tau)$ is reduced to the problem of finding a stable solution for the system of operator equations (4), where operators A_{QE} and A_{TE} designate in this case:

$$A_{QE}Q \equiv \int_0^\tau Q(\chi)\left((1-\cos\psi)\frac{\sin(\sqrt{\alpha_5}(\tau-\chi))}{\sqrt{\alpha_5}} + \sum_{n=1}^{\infty}\int_{\cos\psi}^{1} P_{2n}(z)\,\mathrm{d}z\sum_{r=1}^{3}\Omega^{**}(\gamma)_{2n,r}\sin(\alpha_{2n,r}(\tau-\chi))\right)\mathrm{d}\chi;$$

$$A_{TE}T \equiv \int_0^\tau T(\chi)\frac{1}{2R}\left(\frac{\sin(\sqrt{\alpha_5}(\tau-\chi))}{\sqrt{\alpha_5}} + \sum_{n=1}^{\infty}P_{2n}(0)\sum_{r=1}^{3}\Omega^{**}(\gamma)_{2n,r}\sin(\alpha_{2n,r}(\tau-\chi))\right)\mathrm{d}\chi,$$

and $E(\tau) = \varepsilon_\theta(\gamma,\tau)\cdot(R-h/2)/q_0$.

The algorithm of solving the given system is similar to that described above.

The values of the accepted geometric parameters of the shell correspond to those in work (Vorobiev et al., 1989): $R = 0.2$ m; $h = 0.00175$ m.

The loading area can be defined approximately on the basis of Hertz's theory of static compression of two balls:

$$\psi = \frac{1}{R}\sqrt[3]{\frac{3}{4}m_b g\left(\frac{1-\nu^2}{E}+\frac{1-\nu_b^2}{E_b}\right)\bigg/\left(\frac{1}{R}+\frac{1}{R_b}\right)},$$

where m_b and R_b are the ball mass and radius ($R_b = \sqrt[3]{3m_b/(4\pi\rho_b)} = 0.0167$ m at $m_b = 0.152$ kg); g is the gravitational acceleration; E_b, ν_b, and ρ_b are elastic constants, and the density of the ball material.

As the investigations carried out have shown, the value of ψ, for the investigated range of its change, has no significant effect on the results of solving the inverse problem.

The regularization parameter α (see (6)) used during computation was computed using the residual principle, for which the relative residual factor κ was taken to be 0.05. The results obtained in the form of function $Q(\tau)$, describing the contact force action of the ball on the hemispherical shell, are shown in Fig. 4.

In this Figure, $K = \max(\sum A_{m,\cdot})^{-1}$ ($A_{m,\cdot}$ is the mth row of matrix $A = A_{QE} - A_{TE}(A_{T0})^{-1}A_{Q0}$), and ε_0 is a number used for scaling the source data (see Fig. 5).

Using the solution of the direct problem with the known temporal law of contact loading, the strain changes in time were calculated for other points coinciding with the recording points indicated in work (Vorobiev et al., 1989). The comparison results are shown in Fig. 5.

In the graphs in Fig. 5, the experimental curves are shown as dashed lines, and the design curves are shown as solid ones. In each of the Figures, the distance from the recording points and unit strain computations to the shell pole along the meridian is shown in mm, i.e. from the center of its impact loading.

In Figs. 4 and 5, value $\tau_1 = 0.99$, which was found by treating the experimental results presented in Vorobiev et al. (1989). In so doing, it was taken that $\Delta\tau = 0.0225$.

In finalizing the description of the comparison procedure, it is also necessary to mention the causes, which may lead to inaccuracies in the theoretical and experimental results compared. The first cause is that the identification procedure, in essence, ensures but only an approximation to the exact solution. The second cause is that, when obtaining results for the direct problem of impact interaction of a ball with a shell, a simplified variant of their interaction is used, which corresponds to a constant loading zone and to independence of the contact pressure from the spatial coordinate. The third cause is that, inevitably, there appear inaccuracies in the results obtained by experiment in the form of oscillograms and their treatment, which were used as source data in the identification procedure. These causes appear to be the main ones, though others are possible as well. For instance, the equations in Timoshenko's theory, used for describing the non-stationary behavior of a shell, simulate with a definite accuracy the physically real strain process occurring therein. All these causes lead to inaccuracies in the values of parameters consisting, for instance, in small negative values of the contact pressure (see Fig. 4).

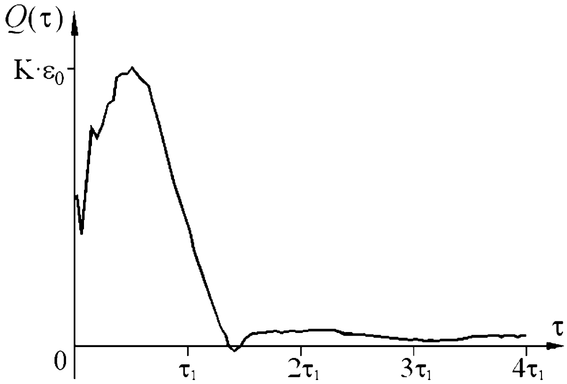

Fig. 4. Result of identifying the contact force interaction.

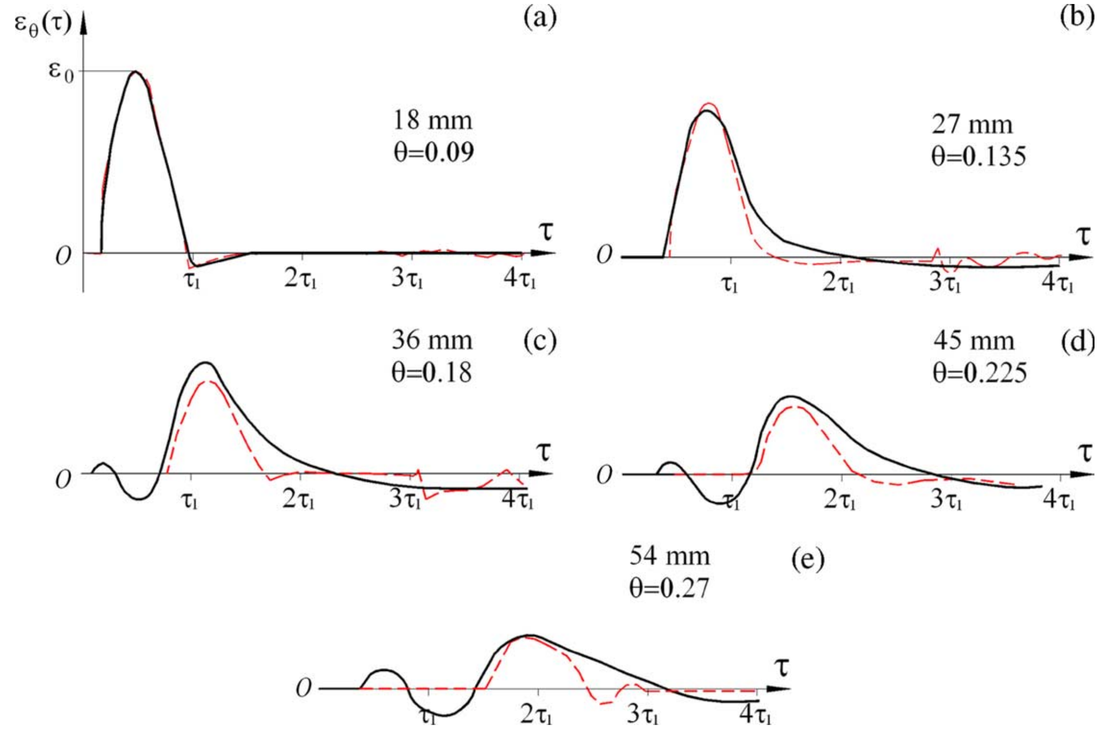

Fig. 5. Comparison of numerical results and experimental data.

6. Conclusion

The paper presents the solution of the problem of identifying the external load acting axisymmetrically on a hemispherical shell with a rigidly fixed edge. The described procedure of solving the problem allows to determine the sought for force loading at different apex angles ψ.

The graphs shown in Figs. 3 and 5 prove the effectiveness of the developed method of solving the given inverse problem, including the use of experimental data.

From the above presentation, it follows that the numerical results of solving the inverse problem depend significantly on the proper selection of the relative residual factor κ (see (7)). Provided information on the degree of discrepancy of accurate source data $\bar{\varepsilon}$ and "noisy" data $\tilde{\varepsilon}$ as $\|\bar{\varepsilon} - \tilde{\varepsilon}\|$ is available, the value of κ_{apt} can be readily computed using the residual principle. But since this value remains unknown, which agrees with current measurement practice, the defining factor is the quality of analyzing the solution of system (6), which satisfies (7).

In conclusion, let us stress the importance and topicality of the problems, one of which has been discussed in this paper. A list of applied problems, which can be solved by using the results obtained during the solution of problems of such kind, is extensive. To date however there is but only a small number of works dedicated to the problem dealt with in this paper, especially in the field of dynamics.

References

Filin, A.P., 1987. Elements of the Theory of Shells. Stroiizdat. 384 pp.

Gladwell, G.M.L., 1984. The Inverse Problem for the Vibrating Beam. Proc. Roy. Soc. London A 393 (1805), 277–295.

Krasnobaev, I.A., Potyetiunko, E.I., 1989. Direct and Inverse Problems in Longitudinal Vibrations of Bars.–Numer. and Analyt. Methods of Solving Problems in Eng. Mech. and the Theory of Elasticity. Rostov on Don. pp. 3–16.

Kylatchanov, K.M., 1988. Solving inverse problems in the theory of soft shells under conditions of high strain. Knižn. odbor. a věd. spisů VÚT brně. 120B, 9–20.

Lukianova, V.N., 1985. Defining the axial force and displacements at sudden loading. Izv. VUZov. Mashinostr. (1), 6–9.

Romanenko, V.A., Mushtakov, N.A., Turkevich Ya.M., 1989. Defining the impact force when striking a cylindrical rod having an arbitrary-shaped end. Optimiz. rezhimov i parametrov gorn. i stroit.-dor. mashin. Kar. Politekh. Inst., Karaganda, pp. 25–28.

Tarasiuk, V.S., 1993. The procedure and solution of inverse axisymmetrical problems in forming shells made of nonlinearly-elastic orthotropic materials. Prikl. mekhanika. Dep. with GNTB of Ukraine 14.10.93, No. 1987-Yk93, 11 pp.

Tikhonov, A.N., Goncharsky, A.V., Stepanov, V.V., Yagoda, A.G., 1990. Numerical methods of solving ill-posed problems. Nauka. 230 pp.

Vorobiev, Yu.S., Kolodiazhniy, A.V., Sevriukov, V.I., Yanyutin, Ye.G., 1989. High-rate Strain in Elements of Structures. Nauk. dumka, Kiev. 192 pp.

Yanyutin, E.G., 1993. Impulse Strain in Elastic Elements of Structures. Nauk. dumka, Kiev. 146 pp.

Yanyutin, E.G., Yanchevsky, I.V., 2000. The inverse problem in the theory of elasticity at non-stationary non-axisymmetrical loading of a cylindrical shell. Vestnik Kharkovskogo Gosudarstvennogo Politekhnicheskogo Universiteta (104), 125–129.